

Phys. 428, Lecture 3:

LECTURE	DATE	INSTRUCTOR	TOPIC
1	April 2	PK	Overview: Imaging equation, inverse problem
2	April 9	PK	2D-LSI imaging systems, X-ray physics: formation and interaction
3	April 16	WH	X-ray detection and imaging systems
4	April 23	PK	X-ray computed tomography (CT) systems
5	April 30	PK	Computed tomography: Biomedical applications
6	May 7	PK	Midterm Exam. Contrast, noise, and image quality
7	May 14	LM	Nuclear decay schemes and isotopes
8	May 21	RM	Gamma cameras: components and systems
9	May 28	WH	Tomography in molecular imaging: SPECT scanners
10	June 4	SB	Positron emission tomography (PET) and hybrid PET/CT scanners
11	June 11	PK	Group project presentations

Each student should email at least one question on today's lecture to our TA, Jackie (jackie24@uw.edu) by Friday.

Please include "Phys 428 Lecture 3 Question" in the subject line.

Why X-rays and not χ -rays (Chi-rays)?



Wilhelm Röntgen

1901 Nobel Prize:

Discovery and characterization of X-ray radiation

Nationality: Germany

Institution: Universities of Strassburg Hohenheim,
Giessen, Würzburg, and Munich



Ernest Rutherford

1908 Nobel Prize: Disintegration of the elements

Discovered several nuclear radiations: α β γ

Nationality: New Zealand, United Kingdom

Institution: McGill University

University of Manchester

Summary of Lecture 3: X-ray radiography

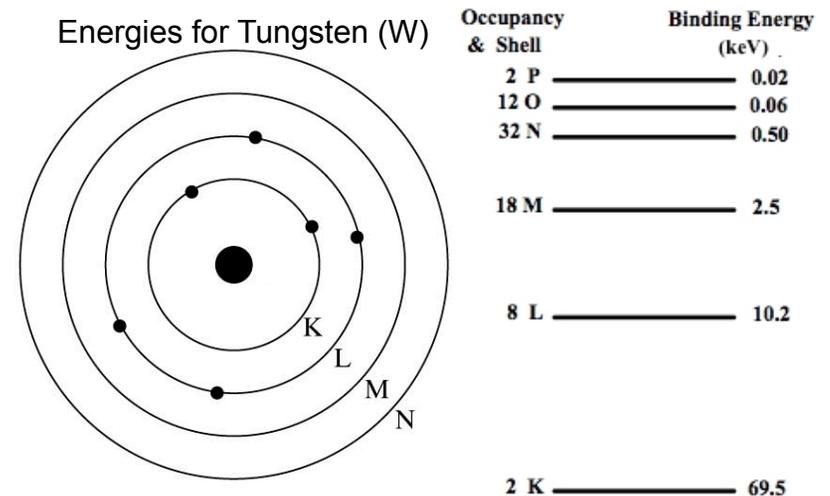
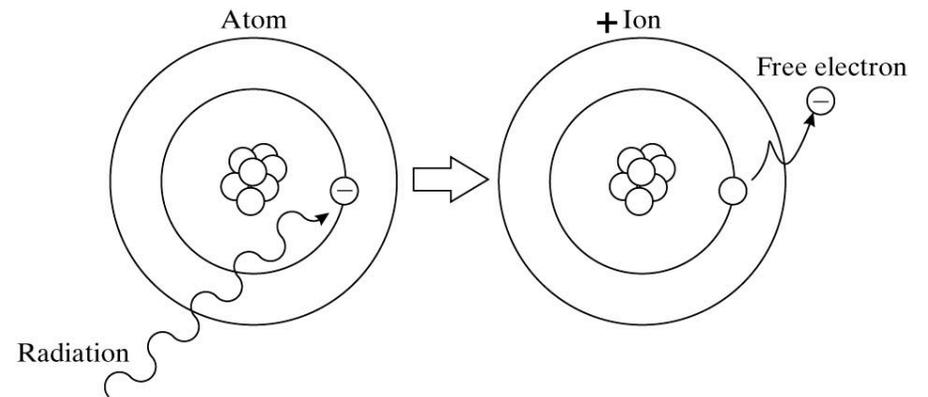
- Ionizing radiation: Properties & Processes
 - ✧ Interaction types
 - ✧ X-ray beam formation
 - ✧ Spectral characteristics
 - ✧ Attenuation and beam hardening
 - ✧ Radiation dose
- X-ray Imaging system: Application
 - ✧ X-ray tube
 - ✧ Beam filtration
 - ✧ Collimation and conditioning
 - ✧ Scatter reduction
 - ✧ Detectors
 - ✧ Imaging Equation
 - ✧ Signal-to-noise ratio
 - ✧ Impact of Scatter
 - ✧ Use of contrast agents
- Computed Tomography ... time permitted

X-ray Radiograph

- **Ion:** Atom or molecule in which the total number of electrons is not equal to the total number of protons, giving it a net positive or negative electrical charge
- **Radiation:** Process in which energetic particles or energetic waves travel through a medium or space
- **X-rays:** Ionizing photon radiation arising from electronic interactions. Can pass through a significant thickness of tissue before being absorbed.
- **Radiograph:** Shadow image related to the projection of material density and composition that can be made with radiation.

Ionizing Radiation

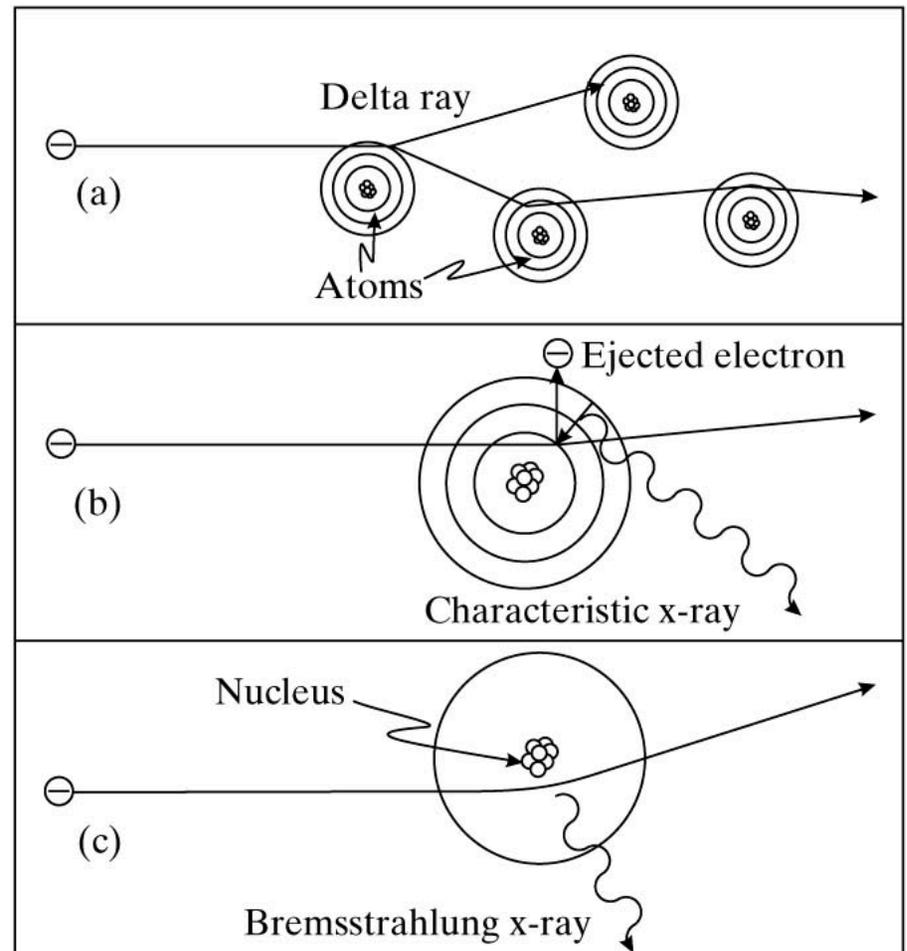
- Radiation (such as high energy electromagnetic photons behaving like particles) that is capable of ejecting orbital electrons from atoms
- Can also be particles (e.g. electrons)
- Ionizing energy required is the binding energy for that electron's shell
- Energy units are electron volts (eV or keV), the energy of an electron accelerated by 1 volt
- For Hydrogen K orbital electrons, $E=13.6 \text{ eV}$
- For Tungsten K orbital electrons, $E=69.5 \text{ keV}$
- In medical imaging we need photons with enough energy to transmit through tissue so are in range of 25 keV to 511 keV and is thus ionizing



Electrons as Ionizing Radiation

- Used in X-ray generation
- Electron kinetic energy $E = (mv^2) / 2$
- Three main modes of interaction in the energy range we are considering

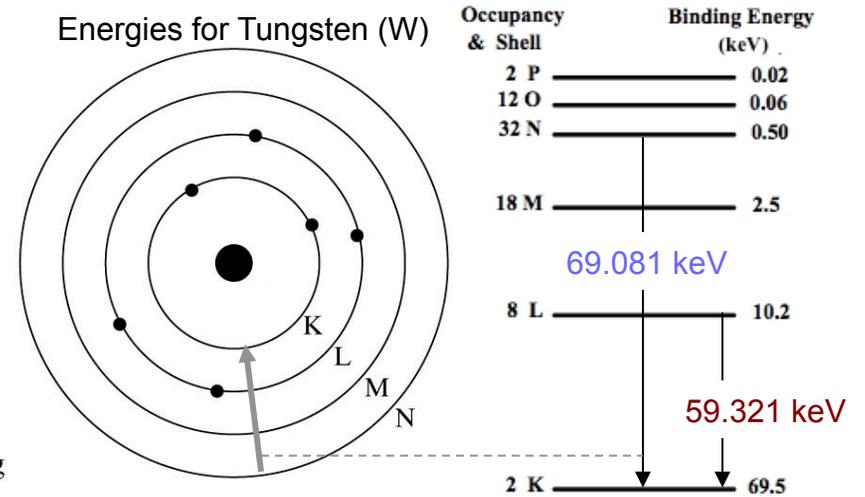
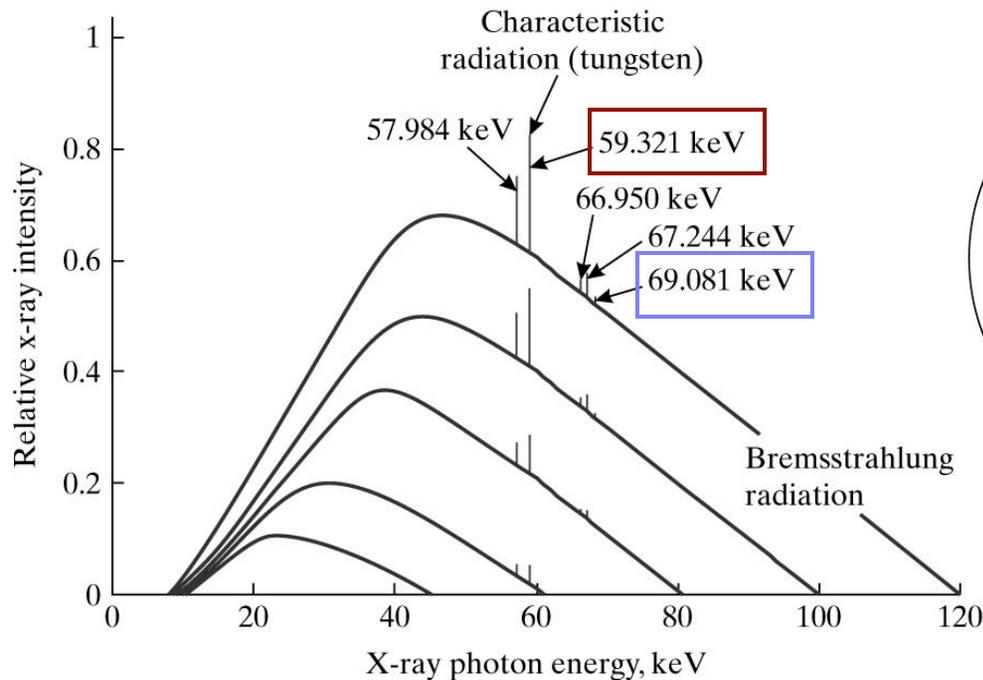
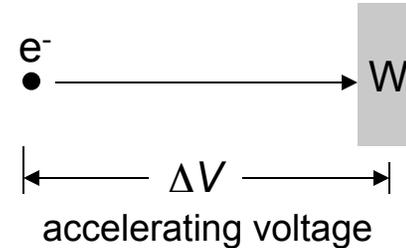
- a) Collision with other electrons and possible creation of delta-rays (high-energy electrons)**
 - This is the most common mode and excited atoms lose energy by IR radiation (heat)
- b) Ejection of an inner orbital electron**
 - This orbit is filled by an outer electron and the difference in energy is released as a 'characteristic x-ray'
- c) Bending of trajectory by nucleus**
 - Since acceleration of a charged particle causes radiation, this causes 'braking radiation' or *bremstrahlung*



X-ray Spectrum from Electron Bombardment

When high energy electrons hit tungsten (symbol W), three effects occur

1. Heat (> 99.9% of the energy)
2. Characteristic x-rays
3. Bremsstrahlung x-rays



Electromagnetic (EM) radiation and Photons

- The energy of EM radiation is given by $E = \hbar\nu$, where $\nu = c/\lambda$ is the wavelength and $\hbar = 6.626 \times 10^{-34}$ J-s is Plank's constant
- At low energies EM radiation behaves like a wave and is non-ionizing
- At high energies EM radiation behaves like a particle (localized wave packet) and can ionize (i.e. > 13.6 eV)

<u>Source</u>	<u>Type</u>
bremsstrahlung	x-rays
transitions in atomic nucleus	gamma-rays

The Electromagnetic Spectrum			
Frequency Range	Wavelengths	Photon Energies	Description
$1.0 \times 10^5 - 3.0 \times 10^{10}$ Hz	3 km–0.01 m	413 peV–124 μ eV	Radio waves
$3.0 \times 10^{12} - 3.0 \times 10^{14}$ Hz	100–1 μ m	12.4 meV–1.24 eV	Infrared radiation
$4.3 \times 10^{14} - 7.5 \times 10^{14}$ Hz	700–400 nm	1.77–3.1 eV	Visible light
$7.5 \times 10^{14} - 3.0 \times 10^{16}$ Hz	400–10 nm	3.1–124 eV	Ultraviolet light
$3.0 \times 10^{16} - 3.0 \times 10^{18}$ Hz	10 nm–100 pm	124 eV–12.4 keV	Soft x-rays
$3.0 \times 10^{18} - 3.0 \times 10^{19}$ Hz	100–10 pm	12.4–124 keV	Diagnostic x-rays
$3.0 \times 10^{19} - 3.0 \times 10^{20}$ Hz	10–1 pm	124 keV–1.24 MeV	Gamma rays

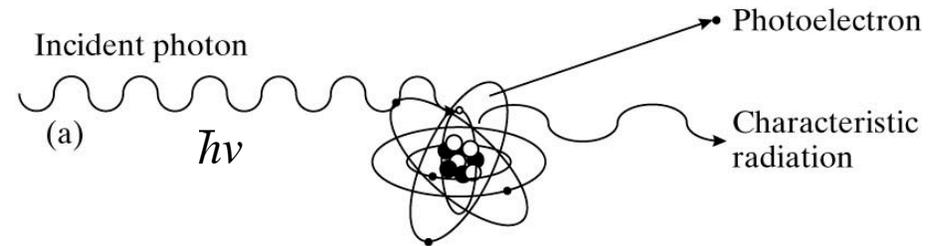
↓ ionizing

Photons as Ionizing Radiation

- Two main modes of interaction in the energy range we are considering

a) Photoelectric effect

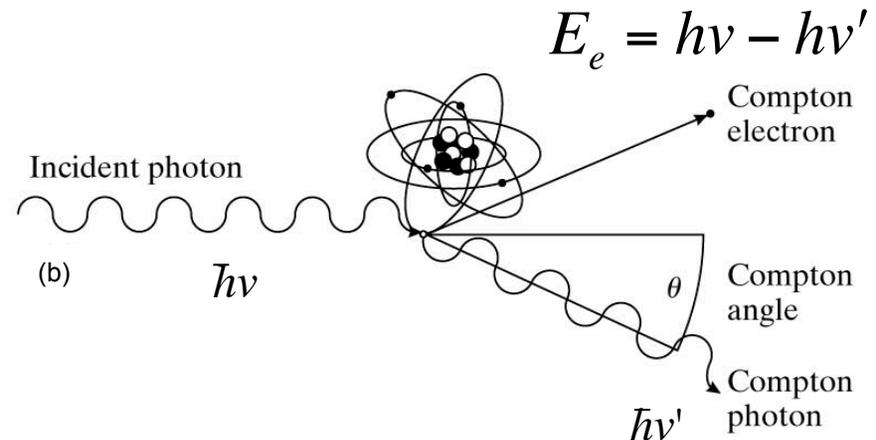
- Causes ejection of an inner orbital electron and thus also characteristic radiation as orbital hole is filled
- Energy of ejected photoelectron is $E_e = h\nu - E_B$



b) Compton scattering

- A photon ejects an outer (valence) electron and the photon scatters off with energy given by:

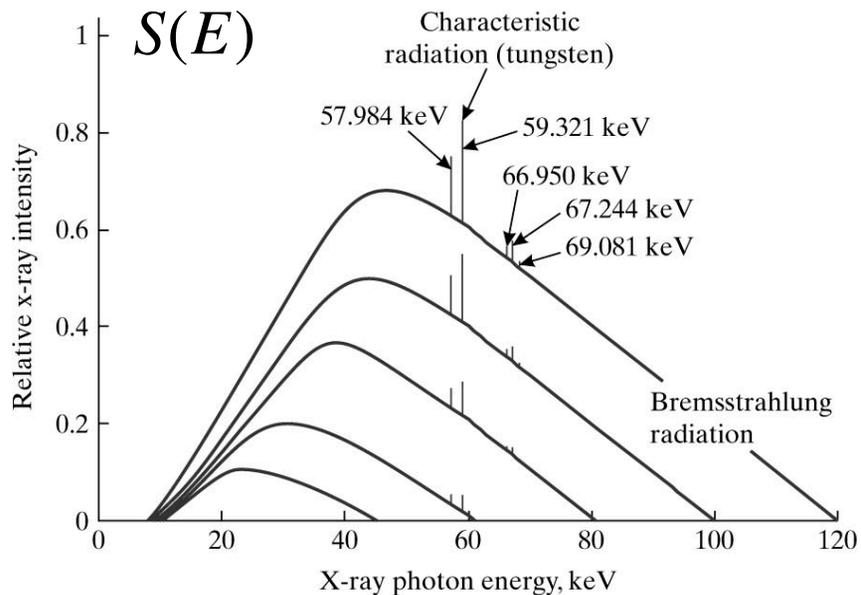
$$\hbar\nu' = \frac{\hbar\nu}{1 + (1 - \cos\theta)\hbar\nu / (m_0c^2)}$$



max E_e ?

X-ray Fluence, Energy, and Intensity

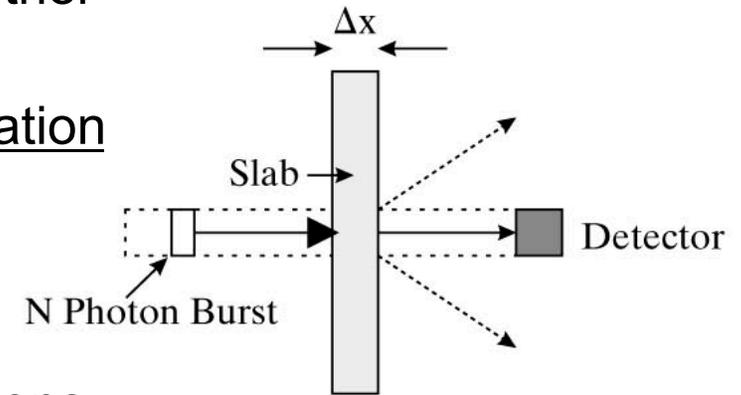
- Fluence rate: number of photons/unit area/time $\phi = N / (A \cdot \Delta t)$
- Beam intensity is the energy fluence rate: $\psi = N\hbar\nu / (A \cdot \Delta t)$
- In X-ray imaging we typically have a continuous range of energies due to bremsstrahlung radiation, so the intensity is the energy weighted integral of the fluence rate



$$I = \int_{E=0}^{E_{\max}} E' S(E') dE'$$

Attenuation

- Photons can be lost due to photoelectric absorption and Compton scatter (and a few other very rare effects)
- We lump these losses together as the attenuation of the x-ray beam
- To analyze, we start with the 'narrow beam' attenuation approximation
- For very thin slabs we expect number of photons lost ΔN to be proportional to Δx and to N , with a proportionality constant, μ , called the linear attenuation coefficient, which is a property of the material at x
- Solving the differential equation gives Beer's law:



$$\Delta N = -\mu N \Delta x$$

$$\frac{d}{dx} N(x) = -\mu(x) N(x)$$

$$N(x) = N(0) e^{-\int \mu(x) dx}$$

fluence



$$I(x) = I_0 e^{-\int \mu(x) dx}$$

intensity

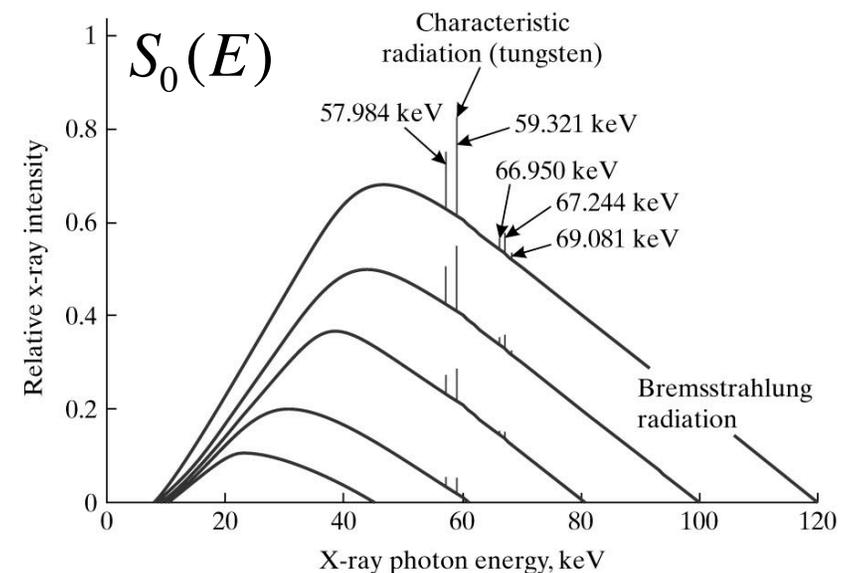
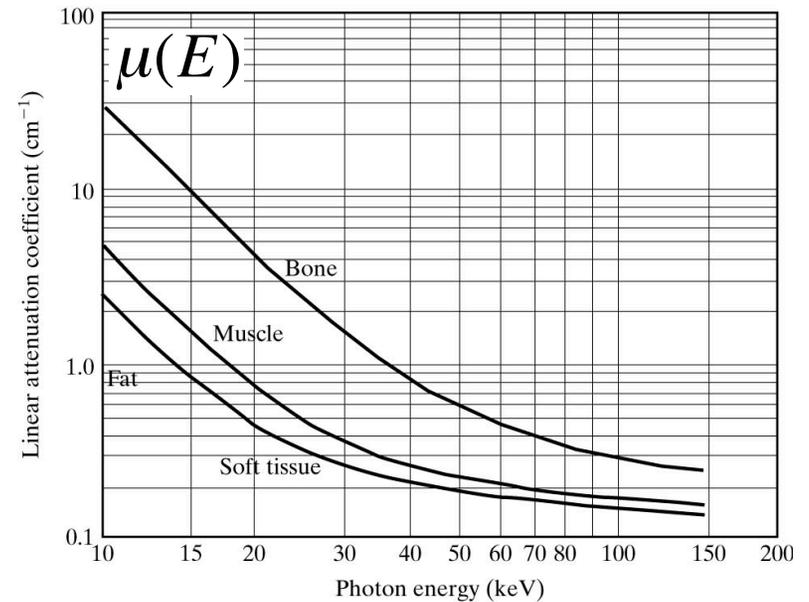
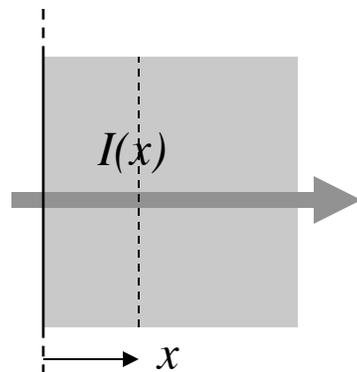
Narrow-beam Polyenergetic Attenuation

- The attenuation depends on material (thus position of material) and energy
- With bremsstrahlung radiation, there is a weighted distribution of energies
- We combine previous results to get the imaging equation

$$I(x) = \int_{E=0}^{E_{\max}} E' S_0(E') e^{-\int_0^x \mu(x', E') dx'} dE'$$

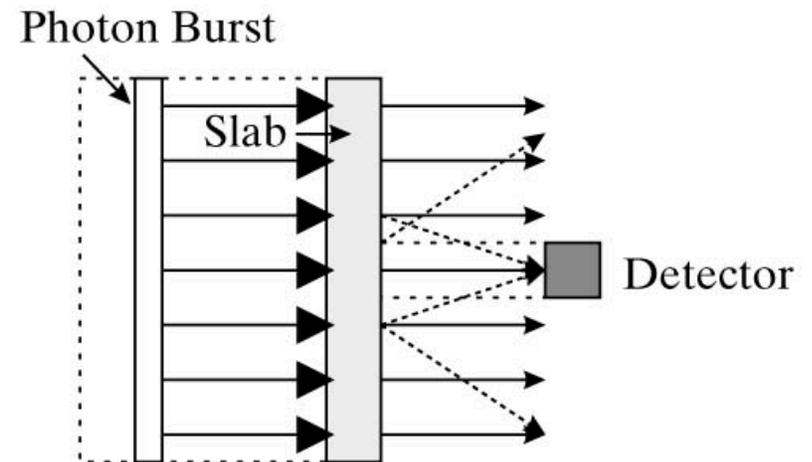
beam intensity along
a line with $\mu = \mu(x)$

$S_0(E)$



Broad-beam Attenuation

- Photons from outside the detector's line-of-sight geometry get scattered toward the detector by Compton interactions
- Further complications come from the energy dependent scatter
- This violates narrow-beam model and should be compensated for by scatter rejection methods, physical or mathematical, if our imaging equation is to remain valid



Radiation Dose from Photons

- Loss of photon energy means some is being transferred to tissue
- Basic concepts:
 - Exposure: number of ion pairs produced in a specific volume of air by radiation
 - Units are coulombs per kilogram of air (C/kg)
 - Useful in medical imaging is the roentgen (R) 2.58×10^{-4} C/kg
 - Can easily be measured with an ionization chamber
 - Absorbed dose: amount of absorbed energy per mass
 - Note this a implicitly a concentration, not a total
 - Units are J/kg, with a special unit of *gray* (Gy)
 - Useful in medical imaging is the rad, which is the absorption of 100 ergs per gram of material
 - 1 roentgen of yields one 1 rad of absorbed dose in soft tissue
 - Equivalent dose: Takes into account type of radiation for tissue T
 - $w_R = 1$ for photons, 2 for protons, 20 for nuclear fragments
 - Effective dose: Takes into account cumulative effect over all tissues
 - meant to compare relative risks between different procedures
 - wildly inaccurate
 - Units are also J/kg, with a special unit of *sievert* (Sv)
 - 1 Gy give 1 Sv for x-rays in soft tissue

$$H_T = \sum_R w_R D_{T,R}$$

$$E = \sum_T w_T H_T$$

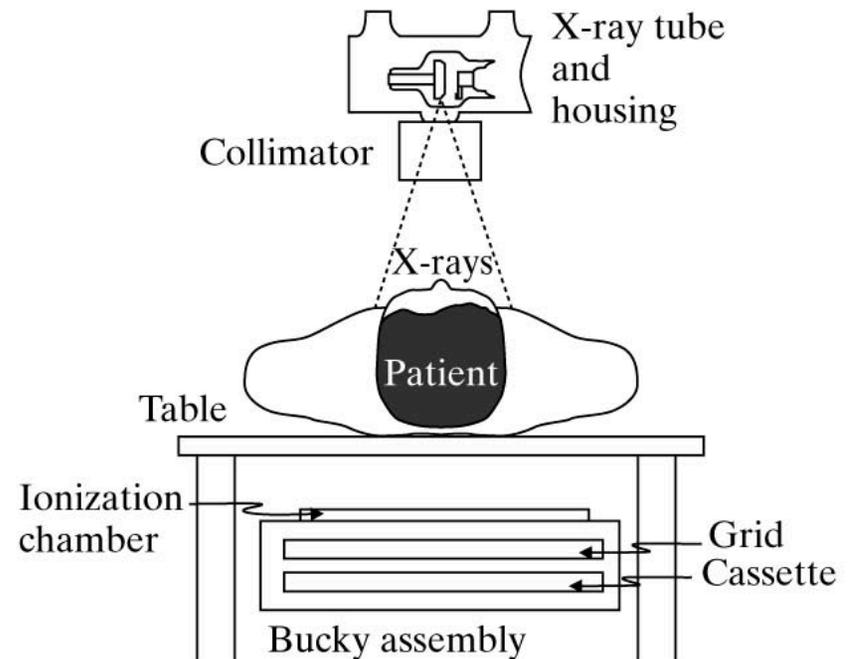
X-ray Imaging Systems

Projection Radiography

- Projection radiography produces radiographs, which are 2-D projections of a 3-D object
- A projection radiography system consists of an x-ray tube, devices for beam filtration and restriction, compensation filters, grids, and (usually) a film-screen detector
- The basic imaging equation describes the energy- and material-dependent attenuation of the x-ray beam produced by the system as it passes through the patient

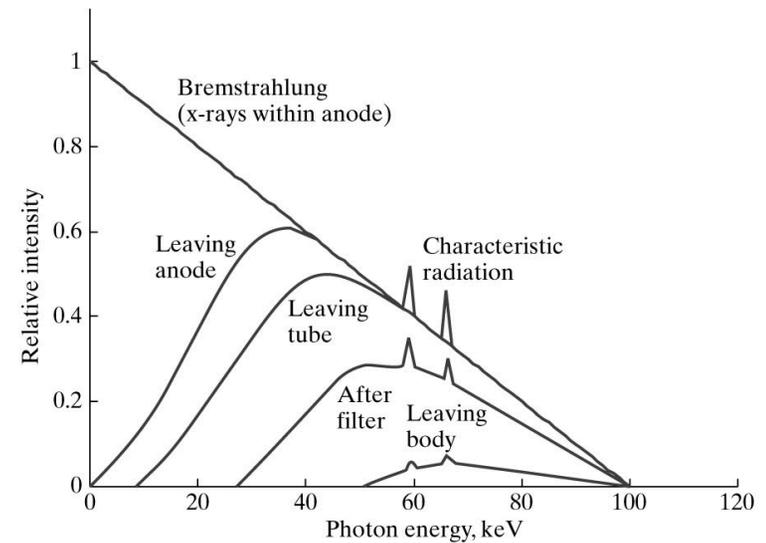
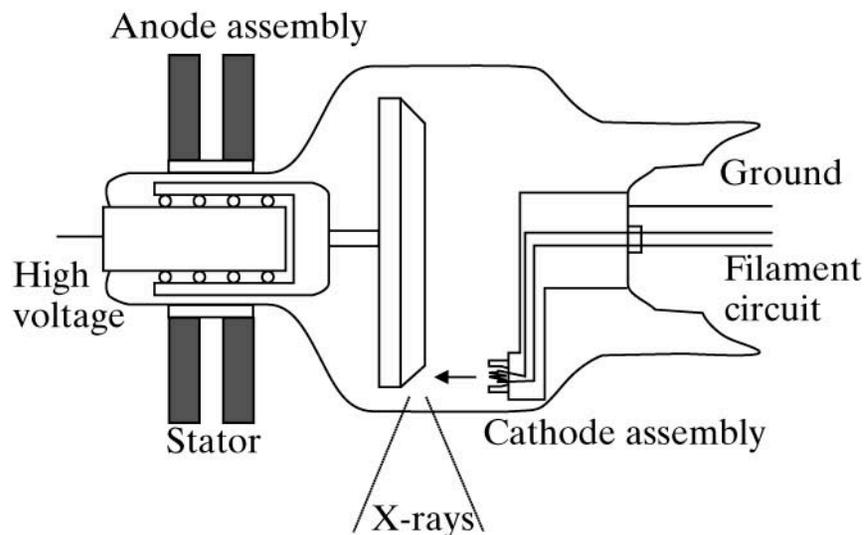
X-ray Imaging Systems

- X-ray source
 - attenuation by object
 - detectors
- } need to be described in the imaging equation
- We want *partial* attenuation



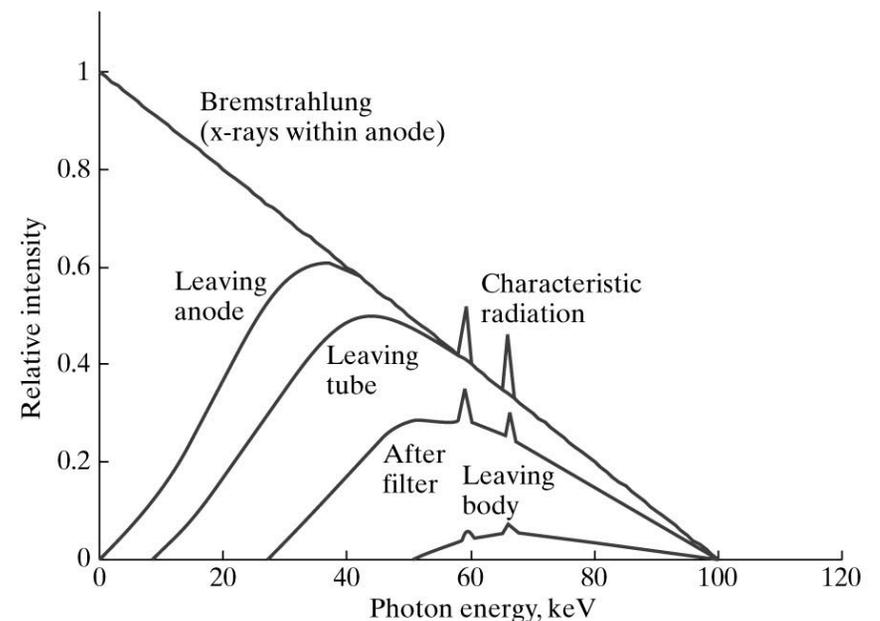
X-ray tubes

- In a vacuum assembly
- A resistive filament is used to 'boil off' electrons in the cathode with a carefully controlled current (10 to 500 mA)
- Free electrons are accelerated by the high voltage towards the anode
- Voltage determines maximum and x-ray energy, so is called the kVp (i.e. kilo-voltage potential), typically 25 kVp to 150 kVp
- High-energy electrons smash into the anode
 - More than 99% energy goes into heat, so anode is rotated for cooling (3000+ RPM)
 - Bremsstrahlung then produces polyenergetic x-ray spectrum



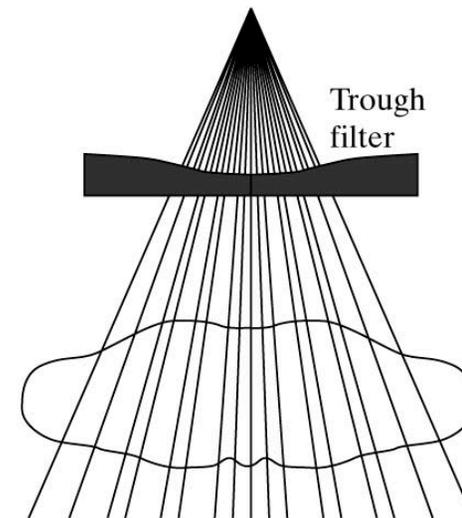
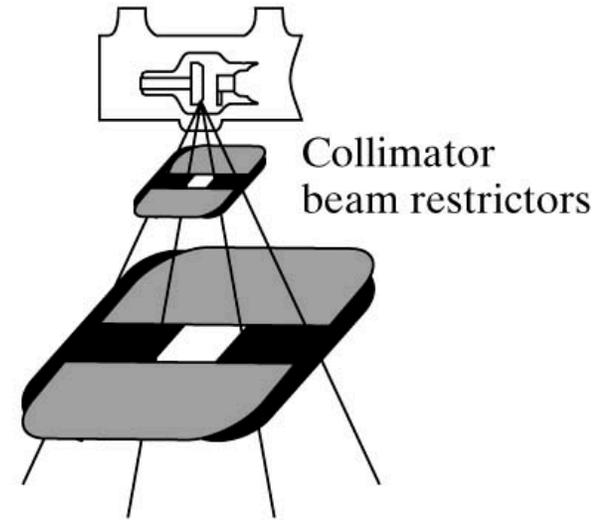
X-ray Beam Filtration

- Lower energy photons are undesirable, as they contribute to radiation dose but not the image quality (if they are all absorbed)
- Some filtering of the low energy photons is provided by the X-ray tube assembly
- A thin (1-2mm) plate of material (Al or Cu) is used to further filter out low energy photons that could not make it through the body
- Note that the average energy increases, a phenomena called *beam hardening*



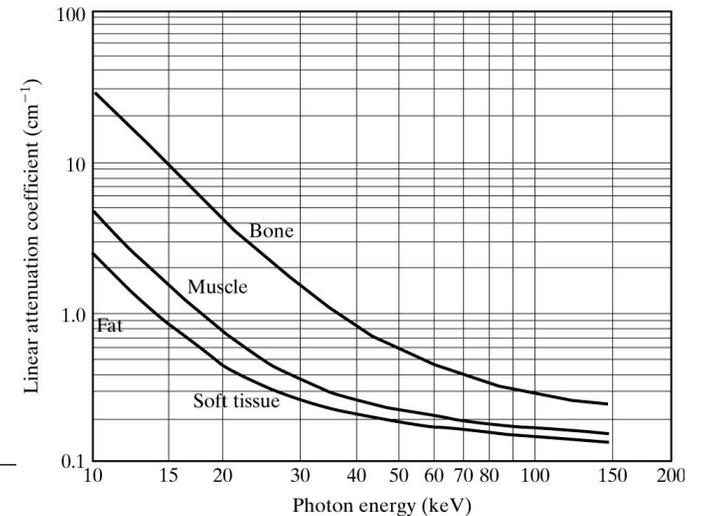
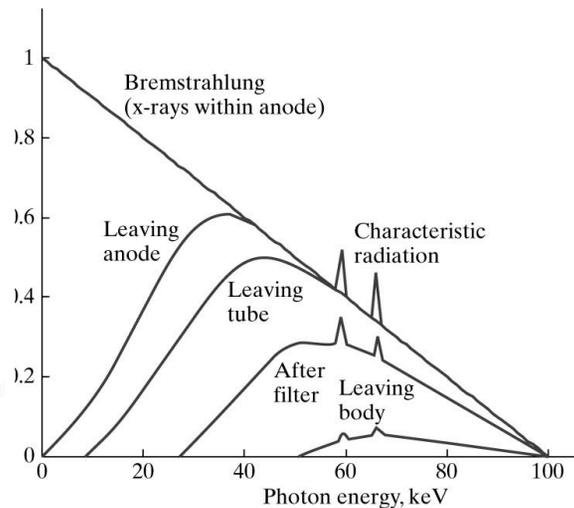
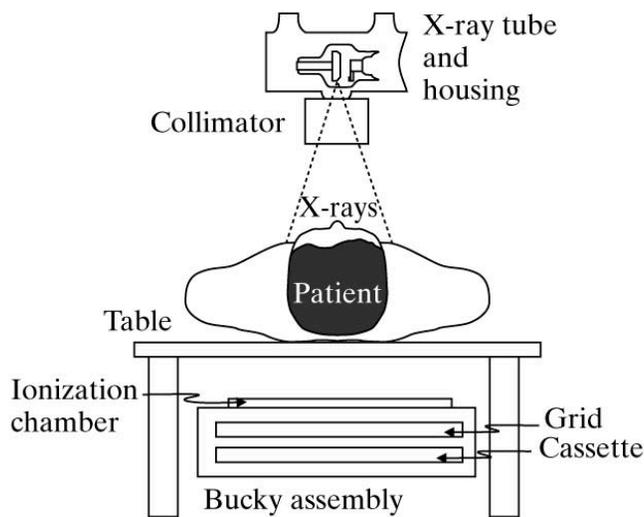
X-ray Beam Collimation and Conditioning

- Many photons are not aimed at the detector, so we can remove them by collimation (also called restriction)
 - high Z materials are used
- It is also possible to adjust the relative fluence to improve the ratio of image SNR to patient radiation dose
 - equalizes the *transmitted* fluence
 - typically use plastic or leaded plastic



Interaction of X-rays in the Body

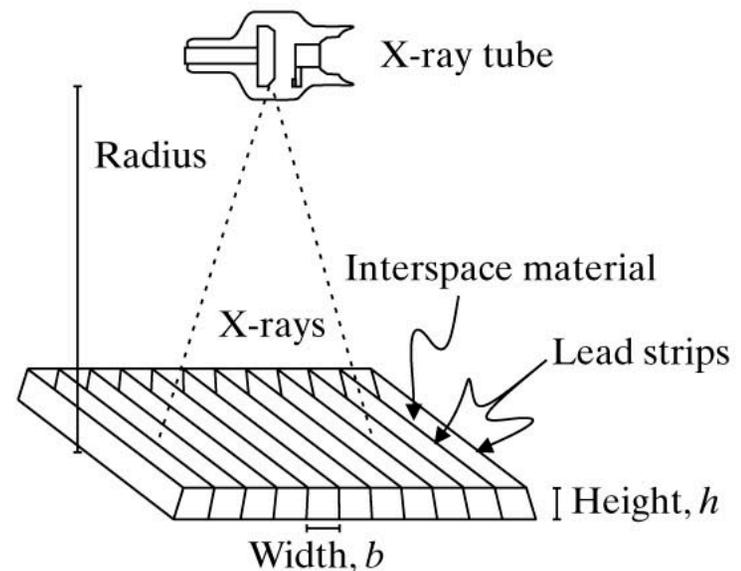
- At this point we have a heavily *conditioned* beam of x-rays entering the body



- The attenuation of x-rays in the body depends on material and energy
- We can enhance attenuation by using 'contrast agents', typically iodine (injected) or barium (ingested)

Scatter Reduction

- Grids can be placed between the patient and the detectors to reduce the amount of scattered radiation detected
- Grids will also reduce the number of un-scattered photons, which is not desirable, so optimization of grid parameters is used improve the ratio of image SNR to patient radiation dose
- Grids are typically moved during the imaging to reduce artifacts

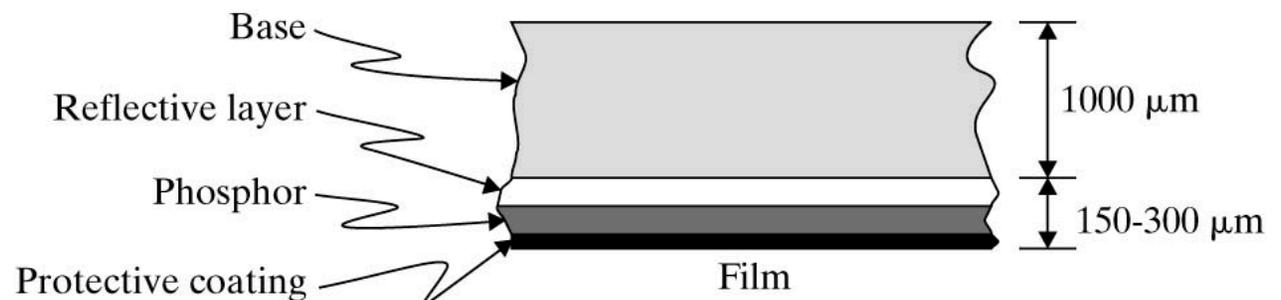


X-ray Detectors

- Several types, including
 - phosphor screen + film systems
 - storage phosphors (also called computed radiography (CR))
 - digital detectors (also called digital radiography (DR))
 - image intensifier + camera systems
- We will only discuss screen+film and DR systems

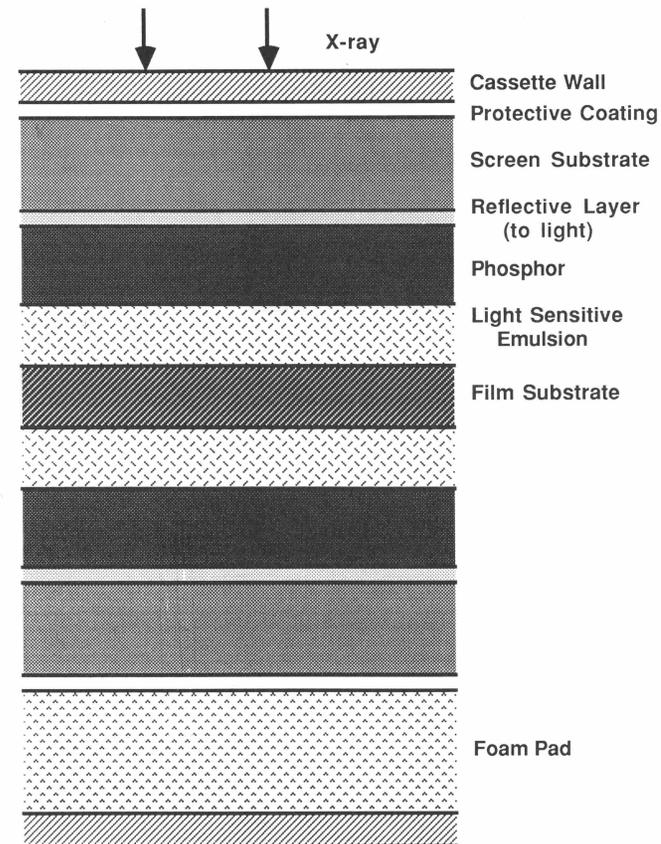
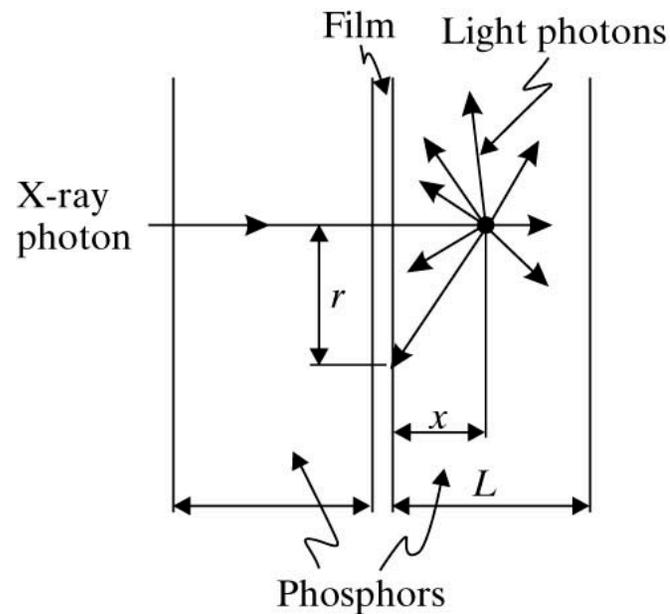
Film-Screen Detectors

- X-rays can expose film directly
 - used by Roentgen in 1895 discovery of mysterious 'x-rays'
 - very inefficient as most rays pass directly through film
- Sensitivity, or 'stopping power' of detectors is a critical feature
 - we want thick and/or dense material
 - another reason why choice of photon energy is important
- Sensitivity of screen film systems can be enhanced by using a luminescent *intensifying screen* on both sides of the radiographic film
- A good intensifying screen phosphor should be highly x-ray attenuating and should emit many light photons for every x-ray photon that is stopped
- In the 1970s rare earth phosphors were introduced with $\sim 10^3$ light photons per incident 50 keV x-ray photon
- Higher conversion efficiencies lead to faster exams and less radiation dose



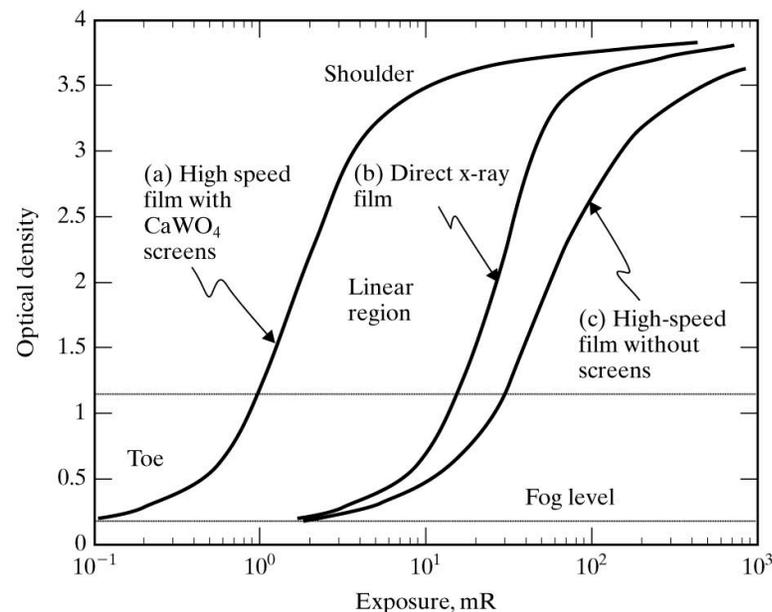
Film Characteristics

- Film cassettes contain double-sided film emulsion and two phosphors to increase sensitivity to x-rays and improve contrast



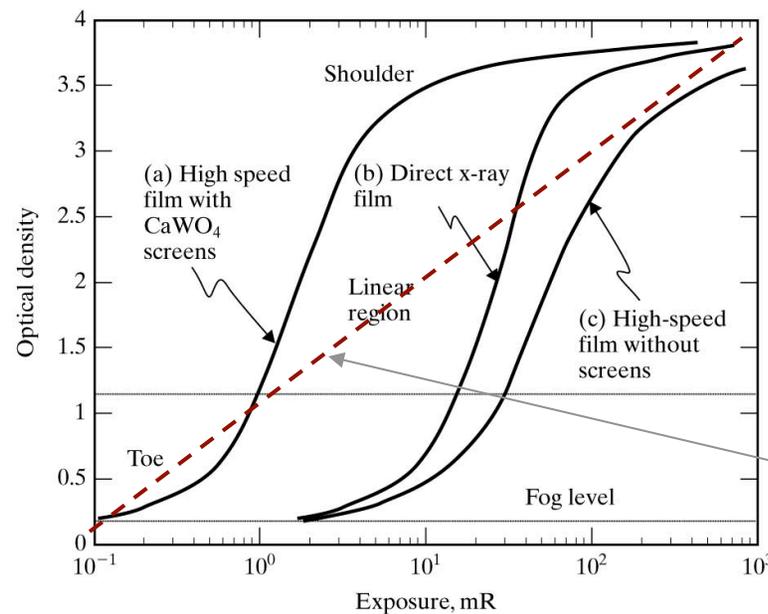
Film-screen H&D Curves

- The overall response of the optical density of the developed film to the x-ray exposure is not linear, but there is a linear region that can be used
- H&D curve is named after Hurter and Driffield
- Many trade-offs in using film, e.g. increasing the slope in the linear region improves contrast, but decreases usable range



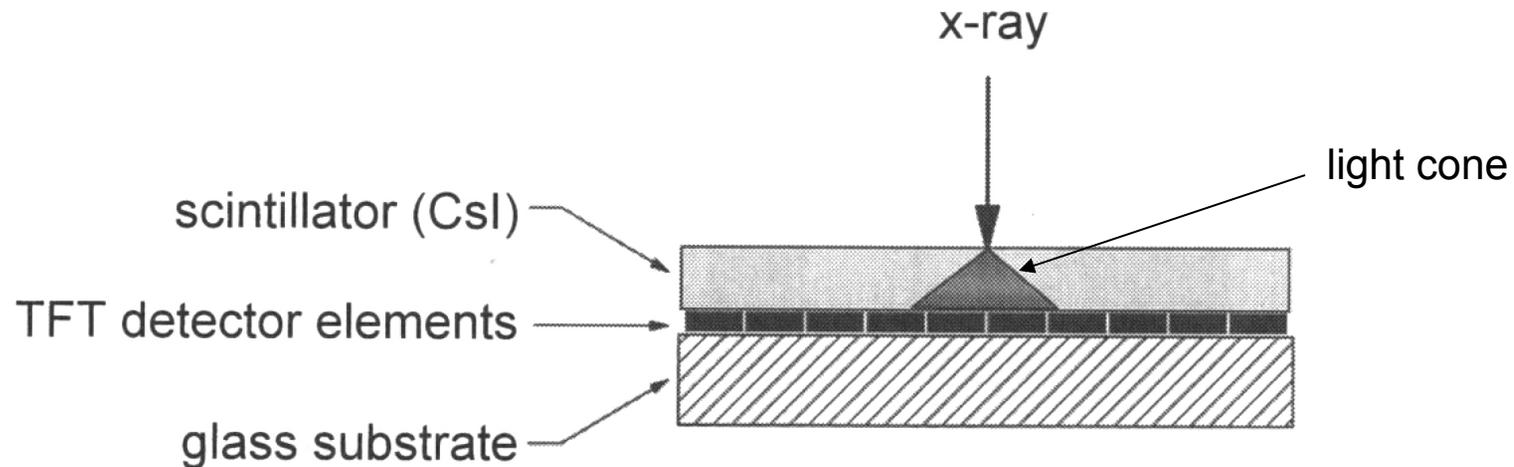
Digital X-ray Detectors

- Screen-film systems are falling out of use for three reasons
 - cost of film (silver based emulsions)
 - digital detectors have a wider dynamic range of linear operation
 - cost of digital detector systems is gradually reducing



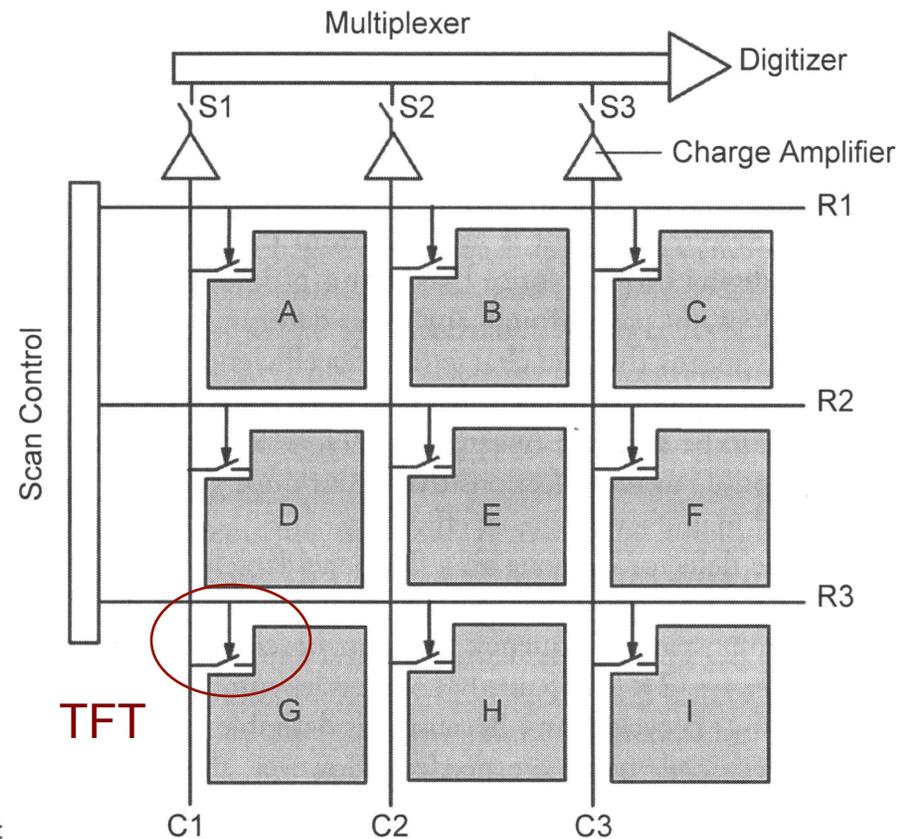
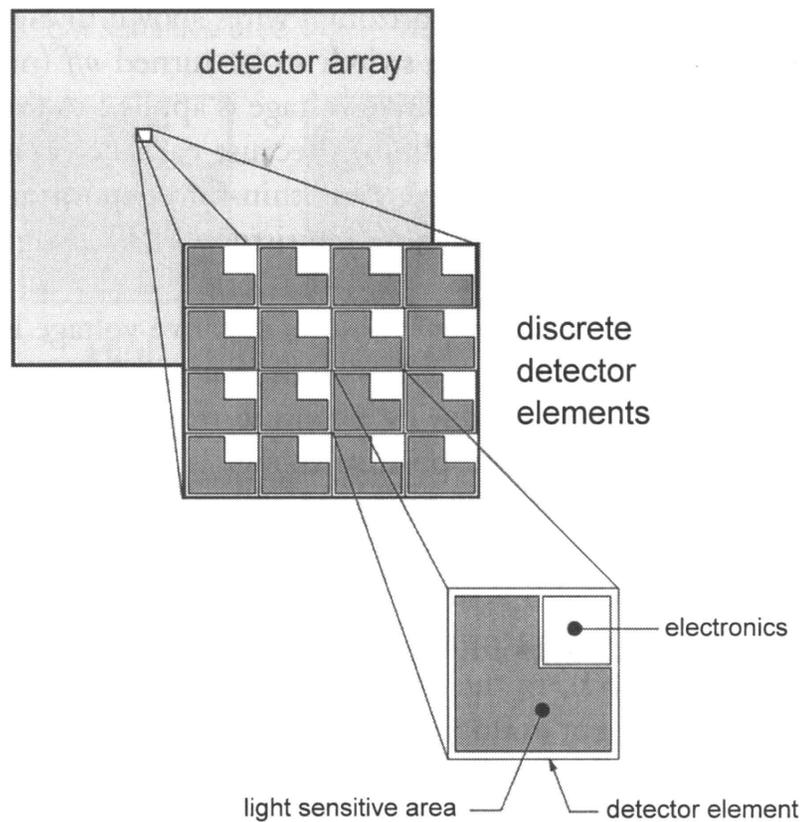
Digital X-ray Detectors

- Digital x-ray detector systems
 - an inorganic 'scintillator', such as CsI, converts x-ray photon to a small flash of optical photons
 - the optical photons hit the surface of a fine 2-D grid of low-noise photodiodes, which can be read out to determine scintillation location



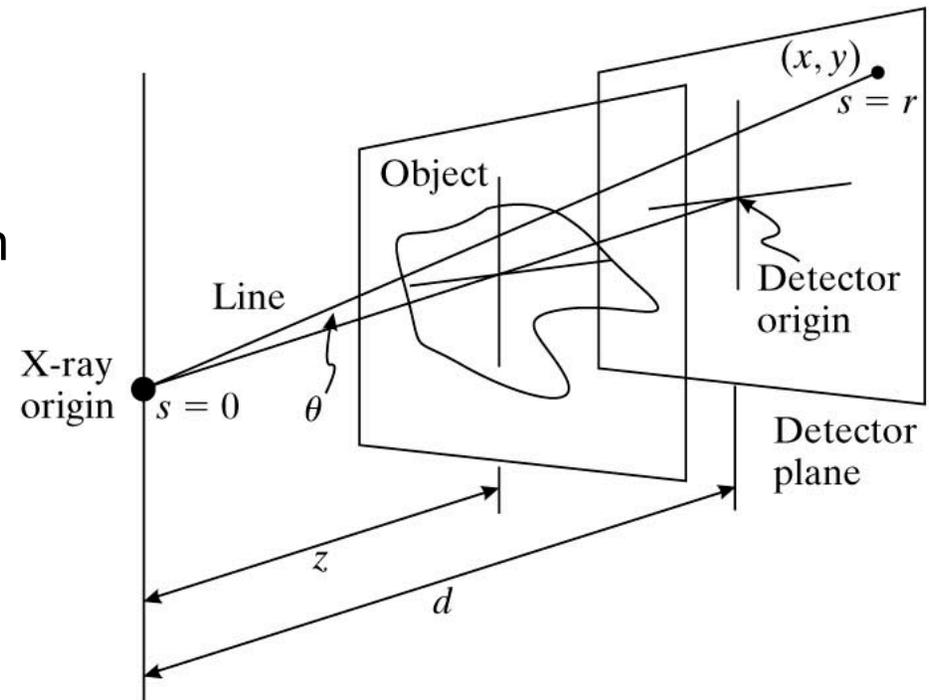
Flat panel digital detectors

- Uses same technology as flat-panel displays



X-ray Imaging Equation

- Recall imaging equation
$$I(x,y) = \int_0^{E_{\max}} E' S_0(E') e^{-\int_0^{r(x,y)} \mu(s,E') ds} dE'$$
- This equation must be modified by several effects
 - inverse square law
 - obliquity
 - beam divergence
 - anode heel effect
 - path length
 - depth-dependent magnification
 - X-ray source size
 - afterglow
 - crosstalk
 - scatter
- We will outline a few effects



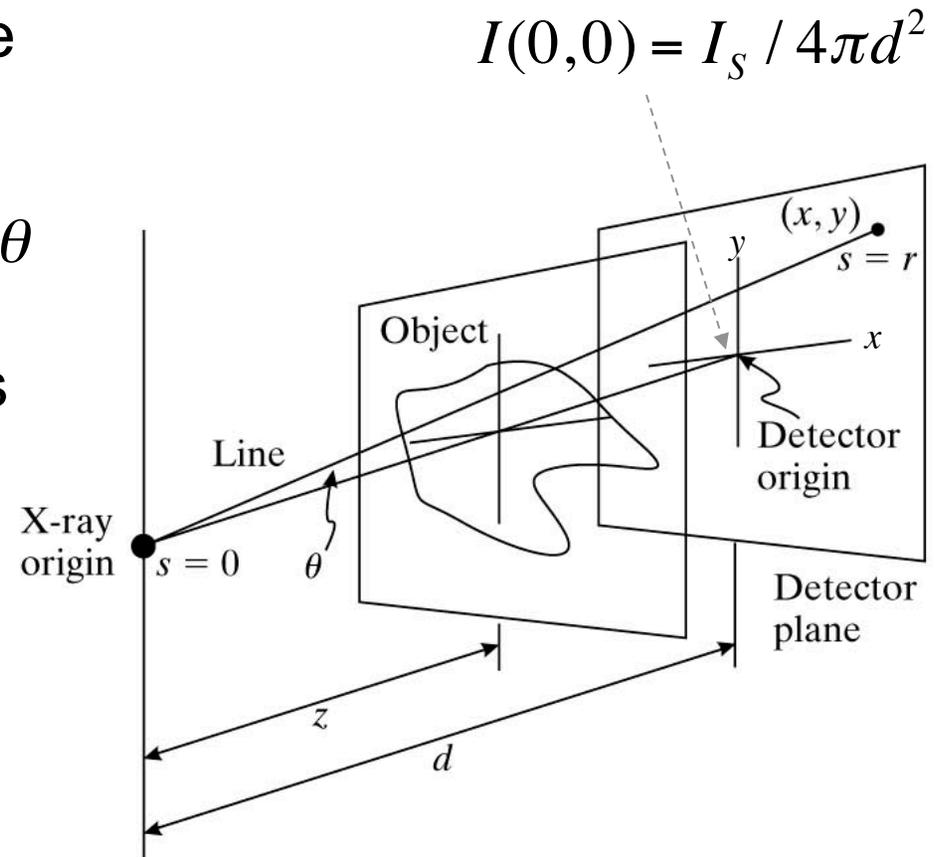
Inverse square law

- the net flux of photons (i.e. photons per unit area) decreases as $1/r^2$, where r is the distance from the x-ray origin

- In general we can modify the imaging equation to

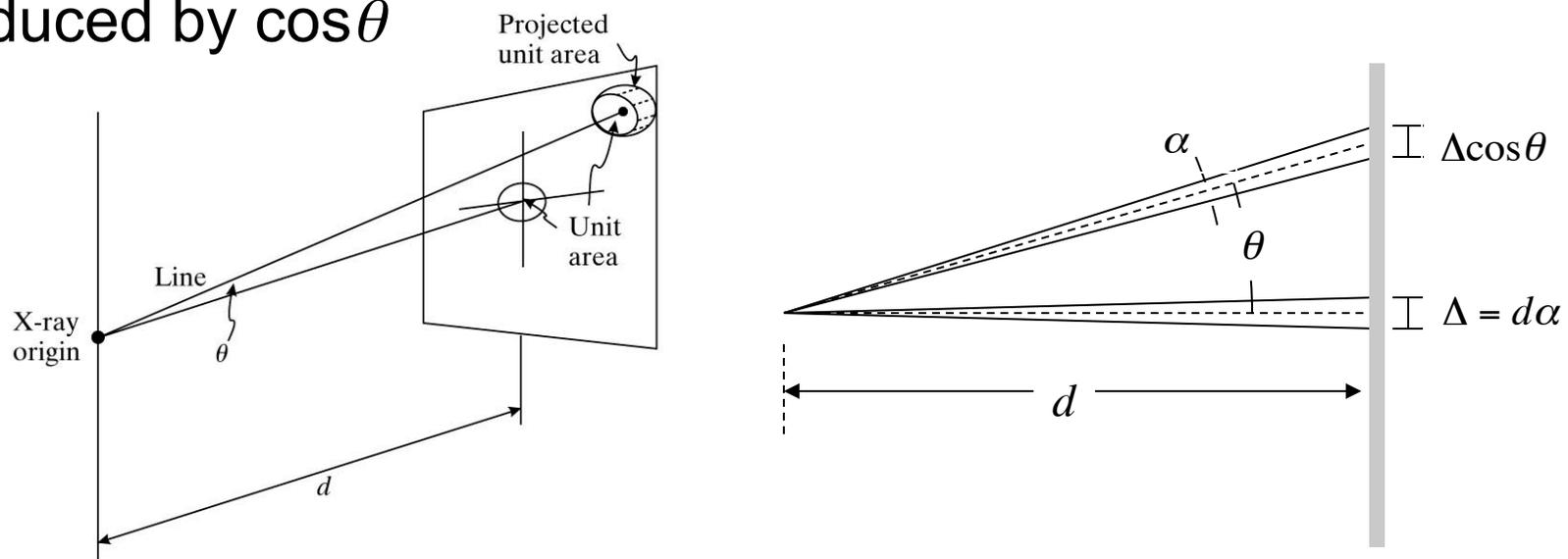
$$I(x,y) = I(0,0) \frac{d^2}{r^2} = I(0,0) \cos^2 \theta$$

- if not taken into account, this effect can lead to incorrect estimate of attenuation



Obliquity

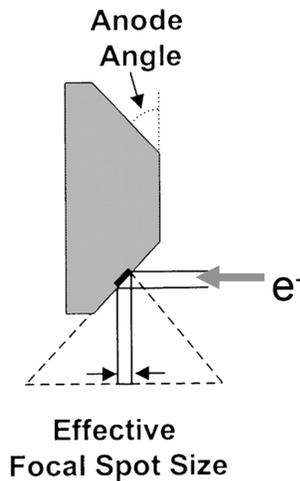
- Away from the origin on the detector plane, the distance from the source is greater ($1/r^2$ effect)
- In addition the orientation of the detector is no longer perpendicular, so the fluence and intensity per unit area is reduced by $\cos\theta$



- Combined effect with $1/r^2$ effect is $I(x,y) = I(0,0)\cos^3\theta$
- If θ is small, this effect can be neglected

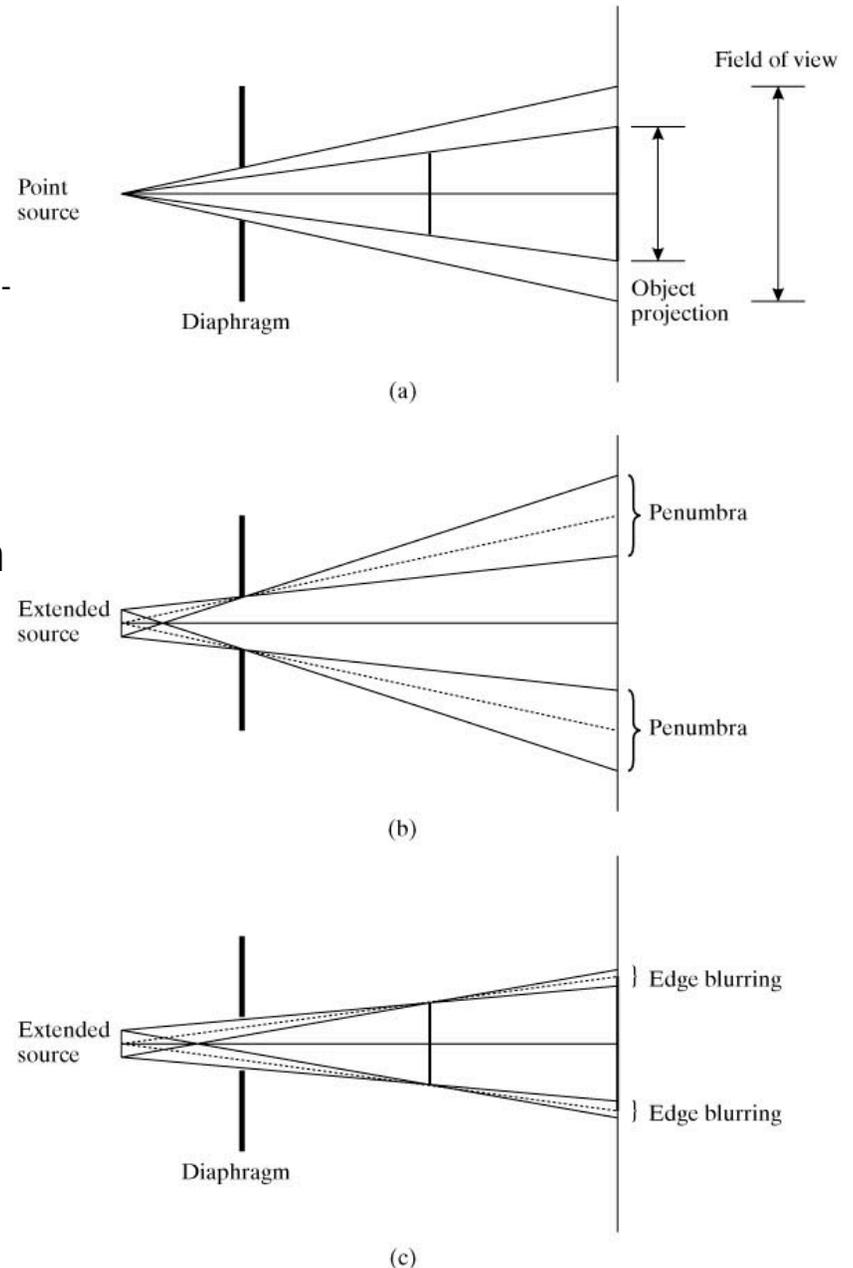
Effects of extended x-ray source

- The extended source or 'spot' on anode for x-ray generation



- This leads to several effects

- Ideal field of view and object projection (with magnification)
- Penumbra at edges of field of view due to extended source
- Blurred object edges due to extended source



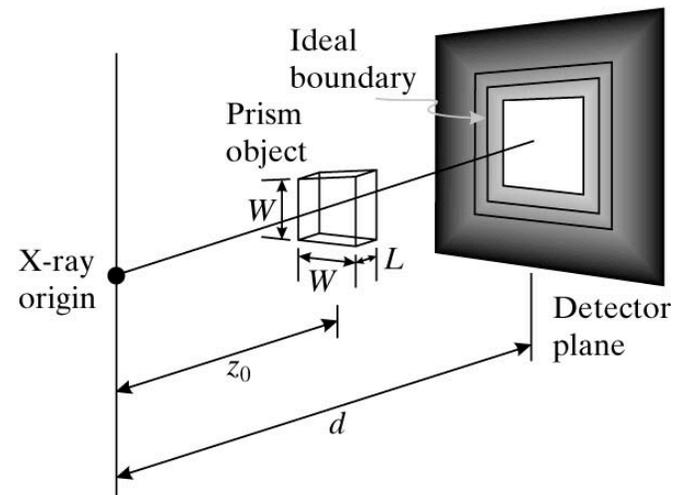
- The extended source effect is a convolution of the source shape with the object shape (with magnification effects also included)

Impact on Imaging Equation

- The imaging equation with the following effects (only)
 - inverse square law
 - obliquity
 - X-ray source size

$$I(x,y) = \overbrace{h(x,y)} * \cos^3 \theta \int_0^{E_{\max}} E' S_0(E') e^{-\int_0^{r(x,y)} \mu(s,E') ds} dE'$$

- The imaging equation should be modified to account for all physics effects
- Some effects can be compensated



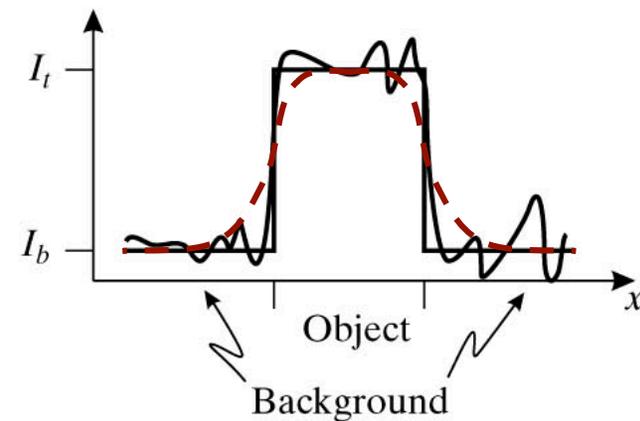
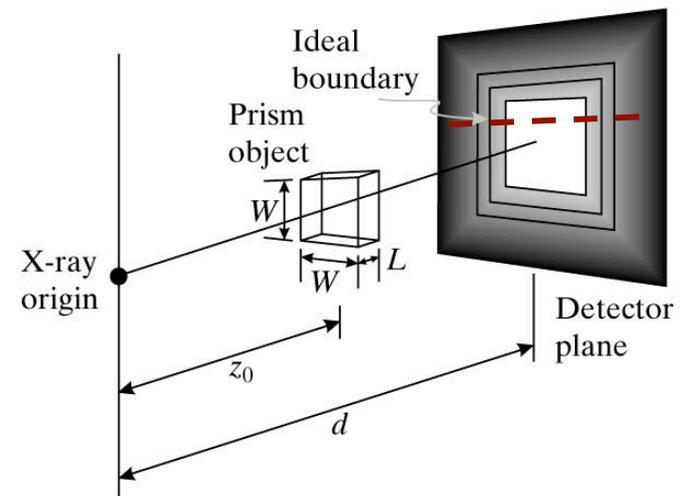
Signal to Noise Ratio

- The imaging equation tells us the measured values, i.e. on a profile through the image
- Physical effects cause distortion of the profile (dashed red curve)
- For now assume a `rect()` profile, with contrast

$$C \triangleq \frac{I_t - I_b}{I_b}$$

- There is also *quantum noise* from the finite # photons for each detector in the background with a signal to noise ratio (SNR) of

$$\text{SNR} = \frac{I_t - I_b}{\sigma_b}$$



Signal to Noise Ratio for X-ray Imaging

- For x-rays with mean energy $h\nu$, the background intensity per detector element is
- Recall variance for a Poisson counting process is equal to the mean
- So SNR is given by $\text{SNR} = C\sqrt{N_b}$
 - TO improve SNR, we need to increase the contrast and/or the # photons
 - As # photons goes up, so does radiation dose
 - Reducing energy of photons increases contrast, noise (fewer transmitted photons), and dose (more absorbed photons)
 - Impact on SNR and SNR/dose ratio is not always clear
- We can group detector elements together to reduce noise, but at some point contrast degrades

$$I_b = \frac{N_b h\nu}{A\Delta t}$$

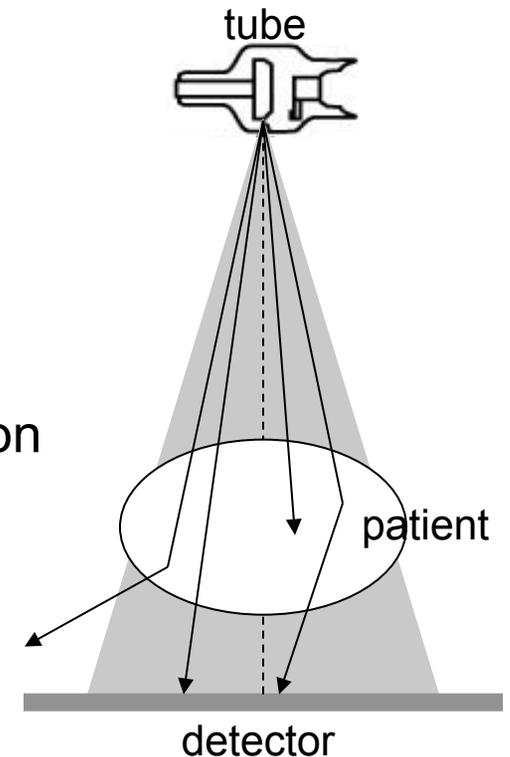
$$\sigma_b^2 = N_b \left(\frac{h\nu}{A\Delta t} \right)^2$$

Impact of Scatter

- Types of events
 - detected true, or primary, photon
 - detected Compton scattered photon
 - undetected Compton scattered photon
 - undetected photon due to photoelectric absorption
- Due to random directions, Compton scatter can be considered a constant

$$C' = \frac{(I_t + I_s) - (I_b + I_s)}{(I_b + I_s)} = \frac{I_t - I_b}{I_b} \frac{I_b}{I_b + I_s} = \frac{C}{1 + I_s/I_b}$$

- So scatter reduces contrast depending on the scatter/true ratio
- In addition scatter increases noise, so there is a further reduction in SNR



Contrast Agents

- Above 40 keV provide a significant enhancement
- There is a very small risk of serious medical complications in the kidney that excretes the contrast agents
- Air can be used to expand the the lungs or GI tract, providing 'negative' or contrast
- Example of an *intravenous pyelogram* used to look for damage to the urinary system, including the kidneys, ureters, and bladder

